Blatt 9

Monday, June 26, 2017 9:51 AM

tips:

Skript S. 65 oder S. 69–71

konst. Koeff. euler

(42) LGS für a, b wann gibt es Lösungen bei welchen λ unendl. Lsg

(43) c) LGS abhängigkeit a=c

(45) c) $t \neq 0$ $x \to ?$

$t = 0$ $x \to ?$

$t \gtrless 0$ $x \to ?$

(41) a) $u''' - 2u'' - 5u' + 6u = 0$

$e^{\lambda x}(\lambda^3 - 2\lambda^2 - 5\lambda + 6) = 0$

$\lambda_1 = 1?$ $1 - 2 - 5 + 6 = 0$

$\lambda_1 = 1$ $\lambda^3 - 2\lambda^2 - 5\lambda + 6 : (\lambda - 1) = \lambda^2 - \lambda - 6$ $\xrightarrow{\text{MNF}}$ $\frac{+1 \pm \sqrt{1^2 - 4\cdot 1 \cdot -6}}{2} = \frac{1 \pm \sqrt{1+24}}{2} = \begin{cases} \frac{1+5}{2} = 3 \\ \frac{1-5}{2} = \frac{-4}{2} = -2 \end{cases}$

$-(\lambda^3 - \lambda^2)$

$-\lambda^2 - 5\lambda$

$-(-\lambda^2 + \lambda)$

$-6\lambda + 6$

$-(-6\lambda + 6)$

0

$\Rightarrow u(x) = c_1 e^x + c_2 \cdot e^{3x} + c_3 \cdot e^{-2x}$ Fundamentalsystem $F\{c_1 e^x, c_2 e^{3x}, c_3 e^{-2x}\}$

b) $x^2 u''(x) - 5xu'(x) + 13u(x) = 0$ homogene euler $\longrightarrow$ $13x^\lambda - 5\cancel{x} \lambda x^{(\lambda-1)} + \cancel{x^2}\lambda(\lambda-1)x^{\lambda-2}$

$u(x) = x^\lambda$

$u'(x) = \lambda x^{(\lambda-1)}$

$u''(x) = \lambda(\lambda-1)x^{\lambda-2}$

$x^\lambda \cdot (13 - 5\lambda + \lambda^2 - \lambda)$

$(\lambda^2 - 6\lambda + 13)$

$\hookrightarrow \frac{6 \pm \sqrt{36 - 4\cdot 1\cdot 13}}{2} = \frac{6 \pm \sqrt{36 - 52}}{2} = \frac{6 \pm \ldots}{}$

allg $u(x) = c_1 \cdot x^{3+2i} + c_2 \cdot x^{3-2i}$

$u(x) = c_1 \cdot x^3 \cos(\ln(x) 2) + c_2 x^3 \sin(\ldots$

d) $4u''''(x) + 4u(x) = 0$

$e^{\lambda x}(\lambda^4 + 4) = 0$

$\lambda^4 + 4 = 0$

$(\lambda^2 - 2\lambda + 2)(\lambda^2 + 2\lambda + 2) = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\frac{2 \pm \sqrt{4 - 4\cdot 1\cdot 2}}{2}$ $\frac{-2 \pm \sqrt{4 - 4\cdot 1\cdot 2}}{2}$

$\frac{2 \pm \sqrt{-4}}{2}$ $\frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm (-2)\cdot i}{2}$

$\begin{cases} 1+i \\ 1-i \end{cases}$ $\begin{cases} -1+i \\ -1-i \end{cases}$

c) $u'''(x) - \frac{3}{x}u''(x) + \frac{7}{x^2}u'(x) - \frac{8}{x^3}u(x) = 0$

$u(x) = x^\lambda$

$u'(x) = \lambda x^{(\lambda-1)}$

$u''(x) = \lambda(\lambda-1)x^{\lambda-2}$

$u'''(x) = \lambda(\lambda-1)(\lambda-2)x^{\lambda-3}$

$x^3 u''' - 3x^2 u'' + 7xu' - 8u(x) = 0$

$\cancel{x^3}\lambda(\lambda-1)(\lambda-2)x^{\lambda-3} - 3x^2\lambda(\lambda-1)x^{\lambda-2} + 7\cancel{x}\lambda x^{\lambda-1} - 8x^\lambda$

$x^\lambda \cdot (\lambda(\lambda-1)(\lambda-2) - 3\lambda(\lambda-1) + 7\lambda - 8)$

(42) a) $x'(t) = 3x(t) - 2y(t)$

$y'(t) = 2x(t) - 2y(t)$ mit $x(t) = a \cdot e^{\lambda t}$, $y(t) = b \cdot e^{\lambda t}$

$x'(t) = 3ae^{\lambda t} - 2be^{\lambda t}$

$y'(t) = 2ae^{\lambda t} - 2be^{\lambda t}$ $\sim \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}$

Gleichsetzen $\lambda ae^{\lambda t} = 3ae^{\lambda t} - 2be^{\lambda t}$, $\lambda be^{\lambda t} = 2ae^{\lambda t} - 2be^{\lambda t}$ $\sim$

$$\begin{array}{ccccc} a & & b & = & \\ \end{array}$$
$\begin{pmatrix} 3a & -2b & = \lambda a \\ 2a & -2b & = \lambda b \end{pmatrix} \rightsquigarrow \begin{pmatrix} 3-\lambda & -2 & | & 0 \\ 2 & -(2+\lambda) & | & 0 \end{pmatrix} \big) - (1+\tfrac{1}{2}\lambda) \rightsquigarrow \begin{pmatrix} 3-\lambda & -2 \\ \frac{\lambda^2}{2} - \frac{\lambda}{2} - 1 & \ldots \end{pmatrix}$

$(3-\lambda)(1+\frac{1}{2}\lambda)$

$3 + \frac{3}{2}\lambda - \frac{2}{2}\lambda - \frac{1}{2}\lambda^2$

$-\frac{\lambda^2}{2} + \frac{\lambda}{2} + 3$

(43) $(1+x^2)u''(x) - 2xu'(x) + 2u(x) = 0$

$0 - 2x \cdot 1 + 2x = 0$ ✓

a) $u_1(x) = x$

$u_1'(x) = 1$

$u_1''(x) = 0$

b) Reduktion der Ordnung

$u(x) = u_1(x) \cdot y(x) = x \cdot y(x)$

$u'(x) = u_1' y + u_1 y' = y(x) + x \cdot y'(x)$

$u''(x) = y'(x) + y'(t) + x \cdot y''(t)$

$(1+x^2)\; u''$ $\quad(-2x)\; u'$ $\quad(2)\; u$

$\rightsquigarrow (1+x^2)(y'(x) + y'(x) + xy''(x)) - 2x(y(x) + xy'(x)) + 2(xy(x))$

$(1+x^2)(y' + y' + xy'') \quad -2x(y + xy') \quad + 2(xy)$

$\underbrace{y' + y'}_{2y'} + xy'' + \underbrace{x^2y' + x^2y'}_{2x^2y'} + x^3y'' - 2xy - 2x^2y' + 2xy$

$xy'' + x^3y'' + y' + y' + \underbrace{x^2y' + x^2y' - 2x^2y'}_{=0} + \underbrace{2xy - 2xy}_{=0}$

$y''(x + x^3) + y' \cdot (2)$

$y''(x + x^3) = y'(-2) \quad | :(y') \quad | :(x+x^3)$

$\frac{y''}{y'} = \frac{(-2)}{(x+x^3)} \quad | \int[\ldots]dx$

partialbruchzerlegung

$\frac{A(x^2+1) + (Bx+C)x}{x(x^2+1)} \sim Ax + (Bx+C)(x^2+1)$

$\frac{\sqrt{-16}}{2} = \frac{6 \pm 4i}{2} = \begin{cases} 3+2i \\ 3-2i \end{cases}$

$x^{3+2i} = e^{\ln(x)(3+2i)}$ $\quad e^{\ln(x)\cdot 3}$ $\quad e^{\ln(x)\cdot 2i}$ $+ \cos(\ln x\, 2) + i \sin(\ln x\, 2)$

$(\ln(x)2)$

$= x^3 + \cos(\ln x 2) + i\sin(\ln x 2)$

(44.)

$\emptyset = x^4 y'''' + 2x^3 y''' + 3x^2 y'' - 3xy' + 4y$

$y(x) = x^\lambda$

$\emptyset = 4x^\lambda - 3x\lambda x^{\lambda-1} + 3x^2\lambda(\lambda-1)x^{\lambda-2} + 2x^3\lambda(\lambda-1)(\lambda-2)x^{\lambda-3} + x \dots$

$\emptyset = 4x^\lambda - 3\lambda x^\lambda + 3\lambda(\lambda-1)x^\lambda + 2\lambda(\lambda-1)(\lambda-2)x^\lambda + \lambda(\lambda-1)(\dots$

$\emptyset = x^\lambda \cdot (4 - 3\lambda + 3\lambda(\lambda-1) + 2\lambda(\lambda-1)(\lambda-2) + \lambda(\lambda-1)(\lambda-2)(\lambda \dots$

$(4 - 3\lambda + 3\lambda^2 - 3\lambda + [(2\lambda^2 - 2\lambda)(\lambda-2)] + [(\lambda^2-\lambda \dots$

$2\lambda^3 - 2\lambda^2 - 4\lambda^2 + 4\lambda \quad [(\lambda^3 - \dots$

$4 - 3\lambda + 3\lambda^2 - 3\lambda + 2\lambda^3 - 2\lambda^2 - 4\lambda^2 + 4\lambda \quad \lambda^4 - \lambda^3 \dots$

$4 + \lambda(-3-3+4-6) + \lambda^2(+3-2-4+ \dots$

$4 + \quad -8\lambda \quad + 8\lambda^2$

$\lambda^4 - 4\lambda^3 + 8\lambda^2 - 8\lambda + 4 : (\dots$

$-(\lambda^4 - 2\lambda^3 + 2\lambda^2)$

$-2\lambda^3 + 6\lambda^2 - 8\lambda$

$-(-2\lambda^3 + 4\lambda^2 - 4\lambda)$

$2\lambda^2 - 4\lambda + 4$

$-(2\lambda^2 - 4\lambda + 4)$

$\emptyset \quad \emptyset \quad \emptyset$

$\rightsquigarrow$ Lösung $\quad y(x) = c_1 x \dots$

$c_2 x \dots$

$c_3 x \dots$

$c_4 x \dots$

$\emptyset \left| \begin{matrix} 0 \\ 0 \end{matrix} \right) \rightsquigarrow$

$a \cdot \left(\frac{\lambda^2}{2} - \frac{\lambda}{2}\right) = 1 \qquad (-2)b + (3-\lambda)a = \emptyset$

$2b = (3-\lambda)\left(\frac{2}{\lambda^2-\lambda}\right)$

$a \cdot (\lambda^2 - \lambda) = 2$

$b = \frac{6}{\lambda^2-\lambda} - \frac{2}{\lambda-1}$

$a = \frac{2}{(\lambda^2-\lambda)}$

$b = \frac{3}{\lambda^2-\lambda} - \frac{1}{\lambda-1}$

$\frac{\lambda^2}{2} - \frac{\lambda}{2} - 1$

$\frac{\frac{1}{2} \pm \sqrt{\frac{1}{4} + 4 \cdot \frac{1}{2} \cdot 1}}{2/2} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 2}$

$\frac{1}{2} \pm \sqrt{\frac{9}{4}} = \frac{1}{2} \pm \frac{3}{2}$

für $a \overset{!}{=} 0$

$\begin{cases} 2 & \lambda_1 \\ -1 & \lambda_2 \end{cases}$

wenn $a = \emptyset$ muss $b = \emptyset$ sein

$\lambda \neq \lambda_1, \lambda_2$

wenn $\lambda = \lambda_1, \lambda_2$

$+2b = (3-\lambda)a \xrightarrow{\lambda=-1} 2b = 3a + a \xrightarrow{\lambda=2} 2b = 3a - 2a$

eines der beiden frei wählbar $\qquad 2b = 4a \qquad b = \frac{a}{2}$

$b = 2a$

$x_1(t) = a \cdot e^{-1t} \qquad y_1(t) = 2ae^{-1t}$

$x_2(t) = a \cdot e^{2t} \qquad y_2(t) = \frac{a}{2}e^{2t}$

$x_{ges}(t) = c_1 \cdot e^{-1t} + c_2 e^{2t} \qquad x(\emptyset) = 3 = c_1 + c_2$

$y_{ges}(t) = 2c_1 e^{-1t} + \frac{c_2}{2} e^{2t} \qquad \frac{1}{2} = 2c_1 + \frac{c_2}{2}$

$\left(\begin{matrix} 1 & 1 \\ 2 & \frac{1}{2} \end{matrix} \left| \begin{matrix} 3 \\ \frac{1}{2} \end{matrix} \right. \right)$

(45) a) $\begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} = A \qquad$ mit $\quad x' = A \cdot x$

$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad x_1' = 3 \dots$

$x_2' = \dots$

(aufteilen)

$v'(x) = A\, v(x)$

$x_1 = \dots$

Ansatz $\quad v(x) = \omega \cdot e^{\lambda x}$

$v'(x) = \omega \lambda e^{\lambda x}$

$x_2 = \dots$

$\omega \lambda e^{\lambda x} = A \cdot e^{\lambda x}$

$\det \begin{pmatrix} 3-\lambda & -2 \\ 2 & -2-\lambda \end{pmatrix} = (3-\lambda)(-2-\lambda) + 4$

$= -6 - 3\lambda + 2\lambda + \lambda^2 + 4$

$= \lambda^2 - \lambda - 2$

also $\quad v(x) = c_1 \cdot e^{2x} \dots$

$v(x) = c_1 \cdot e^{2x} \dots$

b)

$\det \begin{pmatrix} 2-\lambda & -5 \\ 1 & -2-\lambda \end{pmatrix}$

$\left(\begin{matrix} 2+i & -5 \\ 1 & -2+i \end{matrix} \left| \begin{matrix} 0 \\ 0 \end{matrix} \right. \right) \cdot 2+i \dots$

$\left(\begin{matrix} 2-i & -5 \\ 1 & -2-i \end{matrix} \left| \begin{matrix} 0 \\ 0 \end{matrix} \right. \right) \cdot (2-i) \dots$

$(-2-i) \dots$

$-4 + \dots$

$\lambda(\lambda-1)(\lambda-2)(\lambda-3)\,x^{\lambda-4}$

$-2)(\lambda-3)\,x^{\lambda}$

$))$

$)(\lambda-2)(\lambda-3)]$

$\lambda^2-2\lambda^2+2\lambda)(\lambda-3)]$

$-2\lambda^3+2\lambda^2-3\lambda^3+3\lambda^2+6\lambda^2-6\lambda$

$2+3+6)\ +\lambda^3(2-1-2-3)$

$-4\lambda^3 \qquad +\lambda^4$

$\lambda^2-2\lambda+2)=\lambda^2-2\lambda+2$

$\underset{a}{\lambda^2}\underset{b}{-2\lambda}\underset{c}{+2} \qquad \frac{2\pm\sqrt{4-4\cdot1\cdot2}}{2}=\frac{2\pm\sqrt{-4}}{2}\begin{cases}1+i\\1-i\end{cases}$

$1+i$, $1-i$

$=c_1\cdot e^{\ln(x)(1+i)}=c_1 e^{\ln(x)}\cdot e^{i\ln(x)}=c_j\,x\cdot\begin{Bmatrix}\cos(x)\\\sin(x)\end{Bmatrix}\begin{Bmatrix}\cdot\ln(x)\\\cdot 1\end{Bmatrix}$

$c_j\,e^{\ln(x)}\cdot\begin{Bmatrix}Re\\IM\end{Bmatrix}$ {doppelt / einzeln}

$1+i\ \ln x$

$1-i\ \ln x$

$\rightsquigarrow\ y(x)=c_1\cdot x\cdot\cos(x)+c_2\,x\ln x\cos x+c_3\,x\sin x+c_4\,x\ln x\sin x$

$3x_1-2x_2$

$2x_1-2x_2$

$\frac{3}{2}x_1^2-x_2^2$

$=x_1^2-x_2^2$

$\frac{1\pm\sqrt{1+4\cdot2}}{2}=\frac{1\pm3}{2}\begin{cases}\binom{4}{2}\ \lambda_1\\\binom{-2}{2}\ \lambda_2\end{cases}$

$\vec{ev_1}+c_2\cdot e^{-x}\cdot\vec{ev_2}$ eigenVektoren zu $\lambda_{1/2}$

$\begin{pmatrix}3-2&-2\\2&-2-2\end{pmatrix}\left|\begin{matrix}0\\0\end{matrix}\right.\to\begin{pmatrix}1&-2\\2&-4\end{pmatrix}\left|\begin{matrix}0\\0\end{matrix}\right.\ \cdot(-2)\to\begin{pmatrix}1&-2\\0&0\end{pmatrix}\left|\begin{matrix}0\\0\end{matrix}\right.$ $\quad x_2=t,\ x_1=2t \Rightarrow \vec{ev_1}=t\cdot\binom{2}{1}$

$\binom{2}{1}+c_2\cdot e^{-x}\cdot\binom{1}{2}$

$\begin{pmatrix}3+1&-2\\2&-2+1\end{pmatrix}\left|\begin{matrix}0\\0\end{matrix}\right.\to\begin{pmatrix}4&-2\\2&-1\end{pmatrix}\left|\begin{matrix}0\\0\end{matrix}\right.\ \cdot(-2)\to\begin{pmatrix}0&0\\2&-1\end{pmatrix}\left|\begin{matrix}0\\0\end{matrix}\right.$ $\quad x_1=t,\ x_2=2t \quad \vec{ev_2}=t\cdot\binom{1}{2}$

$=(2-\lambda)(-2-\lambda)+5$

$=-4-2\lambda+2\lambda+\lambda^2+5$

$0\doteq\lambda_2+1 \qquad \frac{\pm\sqrt{-4}}{2}=\begin{cases}\lambda_1=i\\\lambda_2=-i\end{cases}$

$\to\begin{pmatrix}2+i&-5\\2+i&-5\end{pmatrix}\left|\begin{matrix}0\\0\end{matrix}\right.\to\begin{pmatrix}2+i&-5\\0&0\end{pmatrix}\left|\begin{matrix}0\\0\end{matrix}\right.$ $\quad x_1=t,\ x_2=\frac{(2+i)t}{5} \quad \vec{ev_1}=t\cdot\binom{5}{2+i}$

$\to\begin{pmatrix}2-i&-5\\2-i&-5\end{pmatrix}\left|\begin{matrix}0\\0\end{matrix}\right.\to\begin{pmatrix}2-i&-5\\0&0\end{pmatrix}\left|\begin{matrix}0\\0\end{matrix}\right.$ $\quad x_1=5t,\ x_2=(2-i)t \quad \vec{ev_2}=t\cdot\binom{5}{2-i}$

$(2-i)$

$2i-2i+i^2=-5$

$v_1=e^{ix}\binom{5}{\ }=\underset{1\times1}{(\cos x+i\sin x)}\underset{2\times1}{\binom{5}{\ }}=\cos x+i\ldots$

c) $v(x)=c_1\cdot e^{2x}\cdot\binom{2}{1}+c_2\cdot e^{-x}\cdot\binom{1}{2}$

$v(0)=\binom{1}{-1}$

$=c_1\binom{2}{1}+c_2\binom{1}{2}$

$v(x)=e^{2x}\binom{2}{1}-e^{-x}\binom{1}{2}$

$$\begin{array}{cc|c} c_1 & c_2 & \\ 2 & 1 & 1 \\ 1 & 2 & -1 \end{array} \quad \circlearrowleft -2 \qquad \begin{array}{cc|c} 0 & -3 & 3 \\ 1 & 2 & -1 \end{array}$$

$\underline{c_2 = -1}$

$c_1 + 2c_2 = -1$

$c_1 - 2 = -1$

$c_1 = 1$

Partialbruchzerlegung

$$\frac{A(x^2+1)+(Bx+C)x}{x(x^2+1)} \sim Ax + (Bx+C)(x^2+1)$$

$$\int \frac{-2}{(x+x^3)}\,dx = -2\cdot\int\frac{dx}{(x+x^3)} = \int\frac{dx}{}$$

$$\frac{-2}{x(x^2+1)} \sim \frac{(x^2+1)(A)+x(Bx+C)}{x(x^2+1)} \sim \frac{\cancel{(x^2+1)}A}{x\cancel{(x^2+1)}} + \frac{\cancel{x}(Bx+C)}{\cancel{x}(x^2+1)}$$

x^2: $A+B=0$
x: $C=0$
x^0: $A=-2 \rightsquigarrow B=2$

$$\rightsquigarrow \frac{A}{x}+\frac{2x}{(1+x^2)}$$

$$\rightsquigarrow \int\frac{-2}{x}+\frac{2x}{(1+x^2)} = -2\int\frac{1}{x}+2\int\frac{x}{(1+x^2)}$$

$$= -2\ln(x) + 2\,\frac{1}{2\cdot 1}\ln(1+x^2) - 0 \sim$$

$$= -2\ln(x) + \ln(1+x^2)$$

$$\frac{y''}{y'} = \frac{(-2)}{(x+x^3)} \qquad \Big|\int[\ldots]\,dx$$

$$\int\frac{y''}{y'} = -2\ln(x)+\ln(1+x^2)+\tilde{c}$$

$$\ln(y'(x)) = \sim \qquad \Big|\,e^x \qquad \text{wähle } c=0$$

$$y'(x) = e^{-2\ln(x)+\ln(1+x^2)+\tilde{c}}$$

$$= e^{-2\ln(x)}\cdot e^{\ln(1+x^2)}$$

$$= e^{\ln(x)^{1\cdot 2}}\cdot e^{\ln(1+x^2)}$$

$$y'(x) = x^{-2}\cdot(1+x^2) = \frac{(1+x^2)}{x^2}$$

$$y(x) = \int\frac{(1+x^2)}{x^2}\,dx$$

$$= \int(1+x^2)\cdot\frac{1}{x^2}\,dx$$

$v \qquad u'$

$$\int\frac{1}{x^2} = \int x^{-2} = \frac{x^{-2+1}}{-2+1} = \frac{x}{-}$$

$$= \int uv - \int uv'$$

$$= -x^{-1}\cdot(1+x^2) - \int -x^{-1}\cdot 2x$$

$$= -\frac{1}{x}\cdot(1+x^2 - \int\frac{-2\cancel{x}}{\cancel{x}}$$

$$= -\frac{1}{x}\cdot(1+x^2)+2x = \frac{-(1+x^2)}{x}+2x$$

$$\rightsquigarrow u(x) = x\cdot y(x)$$

$$= -(1+x^2)+2x^2$$

$$= -1-x^2+2x^2$$

$$= x^2-1$$

c) $u(x) = c_1\cdot x + c_2\cdot(x^2-1) \qquad u(1)=a$

$u'(x) = c_1 + 2c_2\cdot x \qquad u'(1)=b \quad \in\mathbb{R}$

$$u(1) = a = c_1\cdot 1 + c_2\cdot(0) \rightsquigarrow c_1=a$$

$$u'(1) = b = c_1 + 2c_2 \qquad c_2=\frac{b-a}{2}$$

$x_{ges}(t) = c_1 \cdot e \; + c_2 e^{2t}$ $\qquad x(\emptyset) = 3 = c_1 + c_2$

$y_{ges}(t) = 2c_1 e^{-1t} + \frac{c_2}{2} e^{2t}$ $\qquad \frac{1}{2} = 2c_1 + \frac{c_2}{2}$

$$\left(\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & \frac{1}{2} & \frac{1}{2} \end{array}\right)$$

$$\left(\begin{array}{cc|c} 1 & 1 & 3 \\ 4 & -1 & 1 \end{array}\right) -$$

$$\left(\begin{array}{cc|c} 1 & 1 & 3 \\ 3 & 0 & -2 \end{array}\right) \quad 3c_1 = -2 \qquad -\frac{2}{3} + c_2 = 3$$

$c_1 = \frac{-2}{3}$ $\qquad c_2 = 3 + \frac{2}{3}$

$c_2 = \frac{11}{3}$

$-4 +$

$\frac{-1}{1} = -\frac{1}{x}$

$2i - 2i + i^2 = -5$

$$v_1 = e^{ix}\begin{pmatrix}5\\2+i\end{pmatrix} = \underset{1\times 1}{(\cos x + i\sin x)}\underset{2\times 1}{\begin{pmatrix}5\\2-i\end{pmatrix}} = \cos x + i\ldots$$

$$v_2 = e^{-ix}\begin{pmatrix}5\\2-i\end{pmatrix} = (\cos x - i\sin x)\begin{pmatrix}5\\2+i\end{pmatrix}$$

$$v(x) = e^{2x}\begin{pmatrix}2\\1\end{pmatrix} - e^{-x}\begin{pmatrix}1\\2\end{pmatrix}$$

$$v(x) = \begin{pmatrix}x\\y\end{pmatrix}$$

$$V_1 + V_2 = V_1(x)$$

$$\begin{pmatrix}5\cdot(\cos x + i\sin x)\\(2+i)(\cos x + i\sin x)\end{pmatrix} + \begin{pmatrix}5\cdot(\cos x - i\sin x)\\(2-i)(\cos x - i\sin x)\end{pmatrix} = \begin{pmatrix}5\cos x + \cancel{5i\cos x} + 5\cos x - \cancel{5i\cos x}\end{pmatrix} = ($$

$\lambda_1 = i \quad \vec{ev}_1 = \begin{pmatrix}5\\2-i\end{pmatrix}$ $\qquad \lambda_2 = -i \quad \vec{ev}_2 = \begin{pmatrix}5\\2+i\end{pmatrix}$

$\cos x + i\sin x \qquad \cos x - i\sin x$

$$\begin{pmatrix}5\cos x + \cancel{5i\sin x} & + 5\cos x - \cancel{5i\sin x}\\(2\cancel{+i})\cos x + (\cancel{2i}+1)\sin x & + (2\cancel{-i})\cos x - (\cancel{2i}-1)\sin x\end{pmatrix} \rightarrow \begin{pmatrix}10\cos x\\4\cos x + 2\sin x\end{pmatrix} = V_1$$

$$\begin{pmatrix}\cancel{5\cos x} + 5i\sin x & \cancel{-5\cos x} + 5i\sin x\\ \underset{2-i}{(2-i)\cos x} + \underset{2i+1}{(2i+1)\sin x} & -\underset{-2+i}{(2-i)\cos x} + \underset{+2i-1}{(2i-1)\sin x}\end{pmatrix} \rightarrow \begin{pmatrix}10i\sin x\\4i\sin x\end{pmatrix} \rightarrow \begin{pmatrix}10\sin x\\4\sin x\end{pmatrix}$$

$$\boxed{V(x) = C_1\cdot\begin{pmatrix}10\cos x\\4\cos x + 2\sin x\end{pmatrix} + C_2\cdot\begin{pmatrix}10\sin x\\4\sin x\end{pmatrix}}$$

$$c_1 + 2c_2 = -1$$
$$c_1 - 2 = -1$$
$$c_1 = 1$$

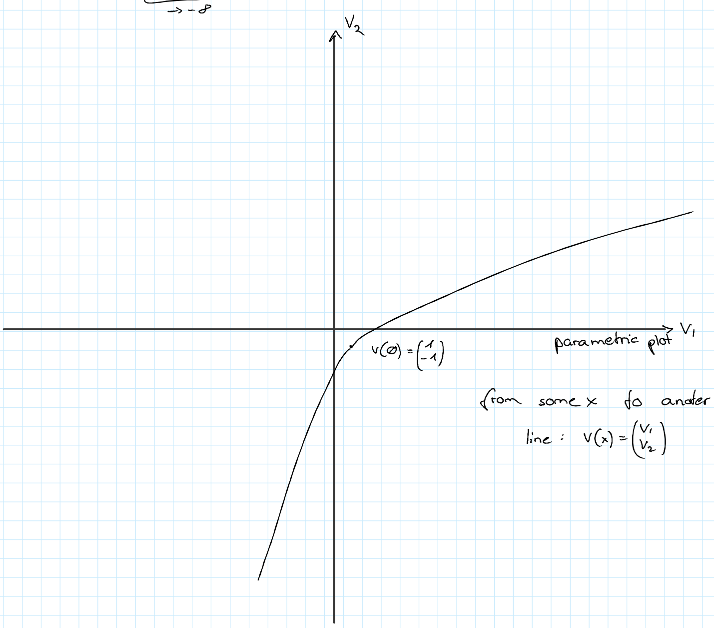
$\begin{pmatrix} 10\cos x \\ 4\cos x \end{pmatrix}$

$$V(x) \xrightarrow{x \to \infty} +\infty$$

$$V(x) \xrightarrow{x \to -\infty} \underbrace{e^{2x}\left(\right)}_{=0} - e^{-x}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\underbrace{-\underbrace{e^{\infty}}_{\to \infty}}_{\to -\infty}$$

$\left. \right) = v_2$



from some x to ander x

line: $v(x) = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$